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Coupling between M2-branes and form fields

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ABSTRACT: In the context of low-energy effective theory of multiple M2-branes, we construct the interaction terms between the world-volume fields of M2-branes and the antisymmetric tensor fields of three- and six-forms. By utilizing the compactification procedure, we show coincidence between the dimensionally reduced coupling and the R-R coupling to D-branes in type II string theory. We also discuss that a cubic term proportional to six-form field reproduces the quartic mass-deformation term in the world-volume theory of multiple M2-branes.

KEYWORDS: M-Theory, Brane Dynamics in Gauge Theories, Conformal Field Models in String Theory

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1 Introduction

To the leading order, the low-energy dynamics of a stack of N parallel D-branes is described by the super Yang-Mills action with $U(N)$ gauge symmetry and the couplings to the bulk fields. In string theory, D-branes carry R-R charges and couple to R-R fields. The form of interaction is given by Wess-Zumino(WZ)-type action [1–3]. Unlike the case of a single Dp -brane where it couples only to R-R fields of rank $p + 1$ or less, a stack of N parallel D-branes couples to all even-form R-R fields in type IIB string theory and to all odd-form R-R fields in type IIA string theory.

Analogous to the D-branes of string theory, in M-theory, we have the M2- and M5-branes and the corresponding three- and six-form fields. About their dynamics, the construction of the world-volume action of multiple M-branes as well as their coupling to the form fields is more difficult than that of multiple D-branes. Recently the world-volume description of low-energy dynamics of multiple M2-branes is available, which is Bagger-Lambert-Gustavsson (BLG) theory with $\mathcal{N} = 8$ supersymmetry and $SU(2) \times SU(2)$ gauge symmetry [4, 5] and Aharony-Bergman-Jafferis-Maldacena (ABJM) theory with $\mathcal{N} = 6$ manifest supersymmetry and $U(N) \times U(N)$ (or $SU(N) \times SU(N)$ gauge symmetry) [6]. Once the action of world-volume fields is obtained, reproduction of its string theory limit is an attractive research direction. The reduction to type IIA string theory of the BLG theory upon circle compactification in the direction transverse to the M2-brane has been achieved in ref. [7, 8].

Despite an overwhelming progress in the understanding of the world-volume action of multiple M2-branes, a little have been done to couple them to the bulk form fields [9, 10]. Therefore, it is intriguing to construct the mutual interaction between the three- or six-form fields and the world-volume fields in the context of BLG and ABJM theories. It is the main objective of this paper to make a proposal for the action which describes the interaction between the M2-branes and the form fields of arbitrary transverse field dependence in the

context of BLG theory and to verify the proposal by reducing it to a similar interaction term in type IIA string theory through circle compactification.

To see the importance of these interaction terms, we make a quick comparison between M2-brane dynamics and the corresponding D2-brane in string theory. In analogy with D2-brane dynamics, in the presence of nonvanishing three-form and dual six-form fields, the low energy dynamics of multiple parallel M2-branes is expected to be described by both their world-volume action, the BLG action S_{BLG} in our case, and the coupling between M2-branes and form fields S_C ,

$$S_{11} = S_{\text{BLG}} + S_C. \tag{1.1}$$

On the other hand, in the type IIA superstring theory the D2-brane action possesses the Dirac-Born-Infeld (DBI) type (or Yang-Mills type in low energy limit) world-volume action of N parallel D2-branes S_{DBI} and the R-R coupling¹ $S_{\tilde{C}}$,

$$S_{10} = S_{\text{DBI}} + S_{\tilde{C}}. \tag{1.2}$$

In nontrivial background, the DBI action is given in terms of the gauge-invariant field strength $\tilde{F}_{\mu\nu} + \tilde{B}_{\mu\nu}$, where $\tilde{F}_{\mu\nu}$ is the field strength of the $U(N)$ gauge field and $\tilde{B}_{\mu\nu}$ is the NS-NS two-form field. In the absence of S_C , it has been verified that, upon circle compactification, the action S_{BLG} reduces to Yang-Mills matter action composed of only $\tilde{F}_{\mu\nu}$. In this paper we show that, after the compactification, the presence of S_C not only produces $S_{\tilde{C}}$ it also gives the missing $\tilde{B}_{\mu\nu}$ piece of the DBI action in the NS-NS background.

Some particular configuration of form fields coupled to D-branes or M-branes can be regarded as the mass deformation of world-volume theories [11, 12]. The (SUSY-preserving) mass deformation of BLG theory is explicitly constructed in [13, 14], which contains the quartic coupling among scalar fields as well as the quadratic mass terms. We show that the WZ-type coupling with particular configuration of form fields reproduces this quartic coupling.

The remaining part of this paper is organized as follows. In section 2 we put forward our proposal for the three- and six-form couplings to multiple M2-branes in BLG theory. Our proposal is made in parallel with the known multiple D2-brane coupling to R-R forms. In section 3 we show that the circle compactification of the action reproduces the corresponding action in ten-dimensional IIA string theory. In section 4 we single out a particular term in the six-form coupling and show that, with a proper choice of the constant background form field, it gives rise to the quartic mass deformation of the BLG theory. Section 5 is devoted to conclusions and discussions.

2 Coupling between M2-branes and form fields in BLG theory

In string theory the coupling of any Dp -brane to R-R form fields $\tilde{C}_{(n)}$ is given by the WZ-type action as [1–3]

$$S_{\tilde{C}} = \mu_p \int_{p+1} \text{STr} \left(P \left[e^{i\tilde{\lambda}_i \tilde{x}^i \tilde{x}} \sum \tilde{C}_{(n)} e^{\tilde{B}} \right] e^{\tilde{\lambda} \tilde{F}} \right), \tag{2.1}$$

¹We put tildes for the fields and the parameters in string theory.

where μ_p is R-R charge of the Dp-brane, $\tilde{\lambda} = 2\pi l_s^2$ is the string scale, \tilde{X} is the transverse scalar field, \tilde{B} is the NS-NS two-form field, and $\tilde{F} = d\tilde{A}$ is field strength of the gauge field \tilde{A} . The trace is taken over the gauge indices, $P[\dots]$ denotes pullback, the summation is taken over all the R-R forms, and i_Φ represents an interior product by Φ^i , explicitly written

$$i_\Phi \tilde{C}^{(n)} = \frac{1}{(n-1)!} \Phi^i \tilde{C}_{ii_1 \dots i_{n-1}} dx^{i_1} \wedge \dots \wedge dx^{i_{n-1}}. \quad (2.2)$$

For later convenience we expand the exponential $e^{i\tilde{\lambda}i_{\tilde{x}}i_{\tilde{x}}}$ and write explicitly the first few terms in the case of $p = 2$,

$$\begin{aligned} S_{\tilde{C}} = \mu_2 \int \frac{1}{3!} dx^\mu \wedge dx^\nu \wedge dx^\rho \text{STr} \left\{ \tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda} \tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\tilde{\lambda}^2 \tilde{C}_{\mu ij} \tilde{D}_\nu \tilde{X}_i \tilde{D}_\rho \tilde{X}_j \right. \\ + \tilde{\lambda}^3 \tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k - 3i\tilde{\lambda}^2 \tilde{C}_{\mu ij} \tilde{X}_i \tilde{X}_j F_{\nu\rho} - 3i\tilde{\lambda}^3 \tilde{C}_{ijk} \tilde{X}_i \tilde{X}_j \tilde{D}_\mu \tilde{X}_k F_{\nu\rho} \\ - i\tilde{\lambda} \tilde{C}_{\mu\nu\rho ij} \tilde{X}_i \tilde{X}_j - 3i\tilde{\lambda}^2 \tilde{C}_{\mu\nu ij k} \tilde{X}_i \tilde{X}_j \tilde{D}_\rho \tilde{X}_k - 3i\tilde{\lambda}^3 \tilde{C}_{\mu ij kl} \tilde{X}_i \tilde{X}_j \tilde{D}_\nu \tilde{X}_k \tilde{D}_\rho \tilde{X}_l \\ - i\tilde{\lambda}^4 \tilde{C}_{ijklm} \tilde{X}_i \tilde{X}_j \tilde{D}_\mu \tilde{X}_k \tilde{D}_\nu \tilde{X}_l \tilde{D}_\rho \tilde{X}_m - \frac{3}{2} \tilde{\lambda}^3 \tilde{C}_{\mu ij kl} \tilde{X}_i \tilde{X}_j \tilde{X}_k \tilde{X}_l F_{\nu\rho} \\ \left. - \frac{3}{2} \tilde{\lambda}^4 \tilde{C}_{ijklm} \tilde{X}_i \tilde{X}_j \tilde{X}_k \tilde{X}_l \tilde{D}_\mu \tilde{X}_m F_{\nu\rho} + \dots \right\}, \quad (2.3) \end{aligned}$$

where \tilde{X}_i ($i = 1, 2, \dots, 7$) are seven transverse adjoint scalar fields with $\tilde{D}_\mu \tilde{X}_i = \partial_\mu \tilde{X}_i + i[A_\mu, \tilde{X}_i]$. Here we omitted the $\tilde{C} \wedge \tilde{B}$ -terms for simplicity. In M-theory we naturally expect a similar coupling between the M-branes (M2- and M5-branes) and the antisymmetric form fields (the three-form field $C_{(3)}$ and the dual six-form field $C_{(6)}$) [15]. Since \tilde{B} is already a part of $C_{(3)}$, $C_{8ij} \sim \tilde{B}_{ij}$ for the compactified eighth direction, the expected action involves the interaction between the form fields and the world-volume fields. In the following, we will consider the BLG theory and construct an analogue of the coupling between the world-volume fields and the form fields. Instead of the original formulation based on three-algebra, we employ a familiar gauge theory formulation [16, 17].

We begin with the BLG theory with eight transverse bi-fundamental scalar fields X_I ($I = 1, 2, \dots, 8$) and two gauge fields A and \hat{A} of $SU(2) \times SU(2)$ gauge symmetry. The bosonic part of the action is

$$S_{\text{bos}} = S_X + S_{\text{CS}} + S_C. \quad (2.4)$$

The first two are well established and are given by

$$S_X = \int d^3x \text{Tr} \left[- (D_\mu X_I)^\dagger D^\mu X_I - \frac{32\pi^2}{3k^2} X_{IJK} X_{IJK}^\dagger \right], \quad (2.5)$$

$$S_{\text{CS}} = \frac{k}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho - \hat{A}_\mu \partial_\nu \hat{A}_\rho - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right), \quad (2.6)$$

where k is the Chern-Simons level and we have used the notation

$$D_\mu X_I = \partial_\mu X_I + iA_\mu - iX_I \hat{A}_\mu, \quad X_{IJK} \equiv X_{[I} X_J^\dagger X_{K]}. \quad (2.7)$$

Due to T-duality between type IIA and IIB string theories, one can restrict possible interaction terms between D-branes and R-R form fields. The exponential factor of the gauge field \tilde{F} in the R-R coupling (2.1) is introduced along the line of open string tadpole computation and is compatible with the T-duality [1–3, 18]. In addition gauge invariance on the D-brane requires $\tilde{B} + \tilde{\lambda}\tilde{F}$ combination and it justifies the exponential factor of the NS-NS two-form field in (2.1). Unlike the superstring theories, however, there seems no concrete guideline for the interactions between M-branes and form fields in M-theory yet. The candidate for the WZ-type coupling between M2-branes and form fields, which is linear in the form fields, is

$$\begin{aligned}
 S_C &= \int_{2+1} \text{Tr} \left(\mu_2 P[C_{(3)}] + \mu'_2 P[(i_X i_X i_X) C_{(6)}] \right) \tag{2.8} \\
 &= \int \frac{1}{3!} d^3x \epsilon^{\mu\nu\rho} \text{Tr} \left\{ \frac{\mu_2}{2} \left[\frac{1}{2} (\hat{C}_{\mu\nu\rho} + C_{\mu\nu\rho}) + 3\lambda C_{\mu\nu I} (D_\rho X_I)^\dagger \right. \right. \\
 &\quad \left. \left. + \frac{3}{2} \lambda^2 \left(\hat{C}_{\mu I J} (D_\nu X_I)^\dagger D_\rho X_J + C_{\mu I J} D_\nu X_I (D_\rho X_J)^\dagger \right) \right. \right. \\
 &\quad \left. \left. + \lambda^3 C_{I J K} (D_\mu X_I)^\dagger D_\nu X_J (D_\rho X_K)^\dagger \right] + (\text{c.c.}) \right. \\
 &\quad \left. + \frac{\mu'_2}{2} \left[C_{\mu\nu\rho I J K} X_{I J K}^\dagger + \frac{3}{2} \lambda \hat{C}_{\mu\nu I J K L} \langle\langle X_{I J K}^\dagger D_\rho X_L \rangle\rangle \right. \right. \\
 &\quad \left. \left. + \frac{3}{2} \lambda C_{\mu\nu I J K L} \langle\langle X_{I J K} (D_\rho X_L)^\dagger \rangle\rangle \right. \right. \\
 &\quad \left. \left. + 3\lambda^2 C_{\mu I J K L M} \langle\langle X_{I J K}^\dagger D_\nu X_L (D_\rho X_M)^\dagger \rangle\rangle \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \lambda^3 \hat{C}_{I J K L M N} \langle\langle X_{I J K}^\dagger D_\mu X_L (D_\nu X_M)^\dagger D_\rho X_N \rangle\rangle \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \lambda^3 C_{I J K L M N} \langle\langle X_{I J K} (D_\mu X_L)^\dagger D_\nu X_M (D_\rho X_N)^\dagger \rangle\rangle \right] + (\text{c.c.}) \right\}, \tag{2.9}
 \end{aligned}$$

where μ_2 is M2-brane tension, $\lambda = 2\pi l_P^{3/2}$ with Planck length l_P , and $\mu'_2 = \beta\lambda\mu_2$. Dimensionless parameter β will be fixed by requiring that when reduced to ten dimensions this action reproduces the correct D2-brane coupling to R-R and NS-NS form fields in type IIA superstring theory. The C and \hat{C} fields in (2.9) are the pullback fields of $C_{(3)}$ and $C_{(6)}$.

For pullback, the eight transverse coordinates are simply replaced with the scalar fields, λX_I and λX_I^\dagger . Since the transverse scalar fields, X_I and X_I^\dagger , in BLG theory transform respectively in bifundamental and anti-bifundamental representations, so do the pullbacked transverse scalar fields,

$$\lambda X_I \rightarrow U(\lambda X_I) \hat{U}^\dagger, \quad \lambda X_I^\dagger \rightarrow \hat{U}(\lambda X_I^\dagger) U^\dagger, \tag{2.10}$$

where U (\hat{U}) is an element of the left (right) $SU(2)$ gauge group. Suppose we have form fields with possible dependence of the transverse coordinates. Then, to maintain the $SU(2) \times SU(2)$ gauge symmetry, the various form fields after the pullback and their gauge transformation properties should appropriately be chosen. When X_I and X_I^\dagger and

their covariant derivatives are multiplied, gauge invariance of every term requires that the bifundamental field and anti-bifundamental field are alternately producted. Then, with the help of trace property, we specifically read the gauge transformation rules of all the pullbacked form fields in (2.9) as

$$\begin{aligned}
 \hat{C}_{\mu\nu\rho} &\rightarrow \hat{U}\hat{C}_{\mu\nu\rho}\hat{U}^\dagger, & C_{\mu\nu\rho} &\rightarrow UC_{\mu\nu\rho}U^\dagger, & C_{\mu\nu I} &\rightarrow UC_{\mu\nu I}\hat{U}^\dagger, \\
 \hat{C}_{\mu IJ} &\rightarrow \hat{U}\hat{C}_{\mu IJ}\hat{U}^\dagger, & C_{\mu IJ} &\rightarrow UC_{\mu IJ}U^\dagger, & C_{IJK} &\rightarrow UC_{IJK}\hat{U}^\dagger, \\
 C_{\mu\nu\rho IJK} &\rightarrow UC_{\mu\nu\rho IJK}\hat{U}^\dagger, & \hat{C}_{\mu\nu IJKL} &\rightarrow \hat{U}\hat{C}_{\mu\nu IJKL}\hat{U}^\dagger, & C_{\mu\nu IJKL} &\rightarrow \hat{U}C_{\mu\nu IJKL}\hat{U}^\dagger, \\
 C_{\mu IJKLM} &\rightarrow UC_{\mu IJKLM}\hat{U}^\dagger, & \hat{C}_{IJKLMN} &\rightarrow \hat{U}\hat{C}_{IJKLMN}\hat{U}^\dagger, & C_{IJKLMN} &\rightarrow UC_{IJKLMN}U^\dagger.
 \end{aligned}
 \tag{2.11}$$

The bracket $\langle i_X i_X i_X \rangle$ denotes interior products by X_{IJK} and its Hermitian conjugate X_{IJK}^\dagger in gauge invariant manner. We also introduced the notation $\langle\langle \dots \rangle\rangle$ to symmetrize objects inside the trace, for instance,

$$\begin{aligned}
 &\langle\langle X_{IJK}^\dagger D_\nu X_L (D_\rho X)_M^\dagger \rangle\rangle \\
 &= \frac{1}{3} \left[X_{IJK}^\dagger D_\nu X_L (D_\rho X_M)^\dagger + (D_\nu X_L)^\dagger X_{IJK} (D_\rho X_M)^\dagger + (D_\nu X_L)^\dagger D_\rho X_M X_{IJK}^\dagger \right].
 \end{aligned}$$

We also note that the different powers of the Planck length l_P in front of some of the terms in the action are chosen based on dimension counting and the $(2\pi)^n$ factors are inserted to mimic the similar factors in ten dimensions.

In ref. [9], the authors proposed the WZ-type couplings in M-theory in terms of 3-algebras with Euclidean and Lorentzian metrics. Some of terms of their proposal for the WZ-type action resemble those in our action (2.3), however, in ref. [9] the authors assumed that the three- and six-form fields do not depend on the transverse scalar fields and so transform trivially under the gauge transformation. To get a gauge invariant action they introduced symmetrized constant tensors originated from symmetrized trace of generators with 3-algebra indices. For a specific representation of the 3-algebra, the symmetrized tensors satisfying the gauge invariance of the action were obtained as functions of structure constant of the 3-algebra. After that, ten-dimensional WZ-type action was obtained by using the Higgs mechanism proposed in ref. [7]. Since the resulting action is composed of constant form fields and symmetric tensors depending on specific representation of 3-algebra, it is not clear how to relate the results to the known WZ-type action (2.1) expressed by U(2)-adjoints.

3 Reduction from M-theory to IIA string theory

In the previous section we constructed the analogue of WZ-type coupling (2.9) between M2-branes and form fields. In this section we shall test and justify the obtained candidate by comparing it with the R-R coupling in string theory (2.3) by reducing it to the ten-dimensional type IIA superstring theory. Specifically we expand the action (2.1) and compare the obtained result (2.3) with the dimensionally reduced WZ-type action of M-theory (2.9).

According to the compactification procedure of ref. [7], we can split the transverse scalars into trace and traceless parts,

$$\begin{aligned} X_i &= \tilde{x}_i + i\mathbf{x}_i, \quad (i = 1, 2, \dots, 7), \\ X_8 &= \frac{v}{2}\mathbf{1} + \tilde{x}_8 + i\mathbf{x}_8, \end{aligned} \quad (3.1)$$

where $\tilde{x}_I = x_I^4 \frac{\mathbf{1}}{2}$, $\mathbf{x}_I = x_I^\alpha \frac{\sigma^\alpha}{2}$ ($\alpha = 1, 2, 3$), and v is a very large vacuum expectation value of $\text{Tr}X_8$. We also introduce

$$A_\mu^\pm = \frac{1}{2}(A_\mu \pm \hat{A}_\mu), \quad (3.2)$$

and then A_μ^- becomes an auxiliary gauge field which we can integrate out using its equation of motion. We can rewrite the covariant derivatives as

$$D_\mu X_I = \tilde{D}_\mu X_I + i\{A_\mu^-, X_I\} \quad \text{with} \quad \tilde{D}_\mu X_I = \partial_\mu X_I + i[A_\mu^+, X_I]. \quad (3.3)$$

In order to consider the type IIA limit for the coupling proposed in (2.9), we have to obtain the form of covariant derivatives for the transverse scalars X_I in the limit. Taking into account the contributions from Chern-Simons term (2.6), the kinetic term for the transverse scalar fields and the WZ-type terms, we will solve the equation of motion for A_μ^- in the limit of a large vacuum expectation value v and large Chern-Simons level k , with a fixed v/k . It turns out that the leading term in the solution to A_μ^- is linear in $1/v$ and we can neglect every term containing A_μ^- unless it is multiplied by v or k . Keeping this in mind we rewrite the covariant derivatives for X_8 and the other transverse scalars X_i as follows

$$\begin{aligned} D_\mu X_8 &= \partial_\mu \tilde{x}_8 + iv \left(A_\mu^- + \frac{1}{v} \tilde{D}_\mu \mathbf{x}_8 \right), \\ D_\mu X_i &= \tilde{D}_\mu X_i. \end{aligned} \quad (3.4)$$

Here we notice that the appearance of A_μ^- in the WZ-type action is only through $D_\mu X_8$, which means it always appears in the combination $(A_\mu^- + \frac{1}{v} \tilde{D}_\mu \mathbf{x}_8)$. A_μ^- in the $S_X + S_{\text{CS}}$ also appears only in this combination as verified by ref. [7]. Therefore, applying the Higgs mechanism, we make a shift of the gauge field $A_\mu^- \rightarrow A_\mu^- - \frac{1}{v} D_\mu^+ x_8$ to eliminate the traceless part of the eighth scalar field \mathbf{x}_8 in the resulting Lagrangian. With this shift the covariant derivative of X_8 becomes

$$D_\mu X_8 = \partial_\mu \tilde{x}_8 + iv A_\mu^-. \quad (3.5)$$

We also adopt the Higgs rule in ref. [8] for the covariant derivatives of the scalars X_i and rewrite them as

$$D_\mu X_i \rightarrow i\tilde{D}_\mu \tilde{X}_i, \quad (D_\mu X_i)^\dagger \rightarrow -i\tilde{D}_\mu \tilde{X}_i, \quad (3.6)$$

where $\tilde{X}_i = \tilde{x}_i + \mathbf{x}_i$ are U(2) adjoint scalars satisfying the following defining properties,

$$\begin{cases} \text{(I)} \quad \tilde{X}_i^\dagger = \tilde{X}_i & : \text{ Hermitian} \\ \text{(II)} \quad \tilde{X}_i \rightarrow U^\dagger \tilde{X}_i U & \text{ for } U \in \text{U}(2) \end{cases} . \quad (3.7)$$

Now the action for the scalar fields (2.5) will take the following simple form

$$S_X = \int d^3x \text{Tr} \left(-\tilde{D}_\mu \tilde{X}_i \tilde{D}^\mu \tilde{X}_i - \partial_\mu \tilde{x}_8 \partial^\mu \tilde{x}_8 - v^2 A_\mu^- A^{-\mu} - V_{\text{bos}} \right) + \mathcal{O}\left(\frac{1}{v}\right), \quad (3.8)$$

where V_{bos} is the potential term. This term will not be affected by the WZ-type action. Therefore, we will not write it explicitly except in the final result. The Chern-Simons action (2.6) also reduces to

$$S_{\text{CS}} = \frac{k}{2\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{Tr} (A_\mu^- \tilde{F}_{\nu\rho}) + \mathcal{O}\left(\frac{1}{v}\right) \quad \text{with} \quad \tilde{F}_{\mu\nu} = \partial_\mu A_\nu^+ - \partial_\nu A_\mu^+ + i[A_\mu^+, A_\nu^+]. \quad (3.9)$$

For our immediate purpose of solving the A_μ^- equation of motion, we write only the part of the WZ-type action that involves A_μ^- explicitly, leaving the remaining part implicit until the end of this section

$$\begin{aligned} S_C = & \int \frac{1}{3!} d^3x \epsilon^{\mu\nu\rho} \text{Tr} \left\{ \frac{\mu_2}{2} \left[\frac{1}{2} (\hat{C}_{\mu\nu\rho} + C_{\mu\nu\rho}) - 3i\lambda C_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\lambda C_{\mu\nu 8} (\partial_\rho \tilde{x}_8 - ivA_\rho^-) \right. \right. \\ & + \frac{3}{2} \lambda^2 (\hat{C}_{\mu i j} + C_{\mu i j}) \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \\ & - \frac{3}{2} i \lambda^2 \hat{C}_{\mu i 8} \left(\tilde{D}_\nu \tilde{X}_i (\partial_\rho \tilde{x}_8 + ivA_\rho^-) - (\partial_\rho \tilde{x}_8 - ivA_\rho^-) \tilde{D}_\nu \tilde{X}_i \right) \\ & + \frac{3}{2} i \lambda^2 C_{\mu i 8} \left(\tilde{D}_\nu \tilde{X}_i (\partial_\rho \tilde{x}_8 - ivA_\rho^-) - (\partial_\rho \tilde{x}_8 + ivA_\rho^-) \tilde{D}_\nu \tilde{X}_i \right) - i \lambda^3 C_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k \\ & + \lambda^3 C_{ijs} \left(\tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j (\partial_\rho \tilde{x}_8 - ivA_\rho^-) - \tilde{D}_\mu \tilde{X}_i (\partial_\rho \tilde{x}_8 + ivA_\rho^-) \tilde{D}_\nu \tilde{X}_j \right. \\ & \left. + (\partial_\rho \tilde{x}_8 - ivA_\rho^-) \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \right) \left. \right] + (\text{c.c.}) \\ & + \frac{\mu_2'}{2} \left[C_{\mu\nu\rho i j s} X_{ij8}^\dagger + \dots + \frac{3}{2} i \lambda \left(\hat{C}_{\mu\nu i j s k} \langle X_{ij8}^\dagger \tilde{D}_\rho \tilde{X}_k \rangle - C_{\mu\nu i j s k} \langle X_{ij8} \tilde{D}_\rho \tilde{X}_k \rangle \right) + \dots \right. \\ & + 3\lambda^2 C_{\mu i j s k l} \langle X_{ij8}^\dagger \tilde{D}_\nu \tilde{X}_k \tilde{D}_\rho \tilde{X}_l \rangle + \dots + \frac{1}{2} i \lambda^3 \hat{C}_{i j s k l m} \langle X_{ij8}^\dagger \tilde{D}_\mu \tilde{X}_k \tilde{D}_\nu \tilde{X}_l \tilde{D}_\rho \tilde{X}_m \rangle + \dots \\ & \left. + \frac{1}{2} i \lambda^3 C_{i j s k l m} \langle X_{ij8} \tilde{D}_\mu \tilde{X}_k \tilde{D}_\nu \tilde{X}_l \tilde{D}_\rho \tilde{X}_m \rangle + \dots \right] + (\text{c.c.}) \left. \right\} + \mathcal{O}\left(\frac{1}{v}\right). \quad (3.10) \end{aligned}$$

For the terms proportional to μ_2' in (3.10), we kept the leading terms proportional to v but neglected all the higher order terms, $\mathcal{O}(k/v, k/v^2, \dots)$. The reason is that, as we pointed out before, $\mu_2' \sim \beta$ and we will show shortly that the numerical factor β is of the order of $1/k$ which is of order of $1/v$.

The variation of the action with respect to A_μ^- gives

$$\begin{aligned} 0 = & \text{Tr} \left\{ \left[-2v^2 A^{-\mu} + \frac{k}{2\pi} \epsilon^{\mu\nu\rho} F_{\nu\rho} + \mu_2 v \lambda \epsilon^{\mu\nu\rho} \left[-\frac{i}{4} (C_{\nu\rho 8} - C_{\nu\rho 8}^\dagger) \right. \right. \right. \\ & + \frac{\lambda}{4} \left((\hat{C}_{\nu i 8} + C_{\nu i 8}) \tilde{D}_\rho \tilde{X}_i + \tilde{D}_\rho \tilde{X}_i (\hat{C}_{\nu i 8} + C_{\nu i 8}) \right) - \frac{i\lambda^2}{12} \left((C_{ij8} - C_{ij8}^\dagger) \tilde{D}_\nu \tilde{X}_i \tilde{D}_\rho \tilde{X}_j \right. \\ & \left. \left. \left. + \tilde{D}_\nu \tilde{X}_i (C_{ij8} - C_{ij8}^\dagger) \tilde{D}_\rho \tilde{X}_j + \tilde{D}_\nu \tilde{X}_i \tilde{D}_\rho \tilde{X}_j (C_{ij8} - C_{ij8}^\dagger) \right) \right] \delta A_\mu^- \right\}. \quad (3.11) \end{aligned}$$

After integrating out A_μ^- , we recall that the two SU(2) groups will be identified and as a result C and \hat{C} become the same. As stated in the paragraph of (2.11), the pullbacked form fields follow bifundamental (anti-bifundamental) or adjoint representation in order to maintain gauge invariance. As in (3.1), the (anti-)bifundamental three- or six-form fields are also decomposed into trace and traceless parts, $C = \check{C}\frac{1}{2} + iC^\alpha\frac{\sigma^\alpha}{2}$. On the other hand, in ten-dimensional type II supergravity, the NS-NS and R-R form fields after pullback are in adjoint representation. Mimicking the case of scalar fields in (3.6)–(3.7) which is consistent with the compactification procedure [7, 8], we naturally define the U(2)-adjoints of pullbacked form fields in ten dimensions by the following linear combination,

$$\tilde{C} = \check{C}\frac{1}{2} + C^\alpha\frac{\sigma^\alpha}{2} = \frac{1}{2}(C + C^\dagger) - \frac{i}{2}(C - C^\dagger). \quad (3.12)$$

Note that the pullbacked form fields in adjoint representation are left as they are. Along this line, we identify the NS-NS two-form field in ten-dimensions from those in eleven-dimensions as

$$\tilde{B}_{PR} = \frac{1}{4}\left[C_{PR8} + C_{PR8}^\dagger - i(C_{PR8} - C_{PR8}^\dagger)\right], \quad (3.13)$$

where we choose 1/4 as a normalization for NS-NS two-form field. Then the quantity inside the inner bracket in (3.11) gives the traceless part of pullback of $\tilde{B}_{\mu\nu}$,

$$\begin{aligned} P[\tilde{B}_{\mu\nu}] &= \tilde{B}_{\mu\nu} + \lambda\tilde{B}_{\mu i}\tilde{D}_\nu\tilde{X}_i + \lambda\tilde{D}_\nu\tilde{X}_i\tilde{B}_{\mu i} \\ &+ \frac{\lambda^2}{3}(\tilde{B}_{ij}\tilde{D}_\nu\tilde{X}_i\tilde{D}_\rho\tilde{X}_j + \tilde{D}_\nu\tilde{X}_i\tilde{B}_{ij}\tilde{D}_\rho\tilde{X}_j + \tilde{D}_\nu\tilde{X}_i\tilde{D}_\rho\tilde{X}_j\tilde{B}_{ij}). \end{aligned} \quad (3.14)$$

On the other hand, since δA_μ^- is traceless, the product of the trace part of $P[\tilde{B}_{\mu\nu}]$ and δA_μ^- is also traceless. Therefore, the A_μ^- equation of motion is simplified as

$$0 = \text{Tr}\left\{\left[-2v^2A_\mu^- + \frac{k}{2\pi}\epsilon^{\mu\nu\rho}\left(F_{\nu\rho} + \mu_2v\lambda\frac{2\pi}{k}P[\tilde{B}_{\nu\rho}]\right)\right]\delta A_\mu^-\right\}. \quad (3.15)$$

Noting that the Yang-Mills coupling constant g_{YM} in the effective action of two D2-branes, the dimensionless string coupling constant g_s , and the string scale $\tilde{\lambda}$ are given by

$$g_{\text{YM}} = \frac{2\pi v}{k}, \quad g_s = g_{\text{YM}}^2 l_s, \quad \tilde{\lambda} = 2\pi l_s^2 \quad (3.16)$$

with $\mu_2\lambda = \frac{1}{4\pi^2 l_p^3} 2\pi l_p^{3/2}$ and $l_p = g_s^{1/3} l_s$, A_μ^- in (3.15) is

$$A_\mu^- = \frac{1}{2g_{\text{YM}}\tilde{\lambda}v}\epsilon_\mu^{\nu\rho}(P[B_{\nu\rho}] + \tilde{\lambda}F_{\nu\rho}), \quad (3.17)$$

where $B_{\mu\nu}$ is the traceless part of $\tilde{B}_{\mu\nu}$.

The gauge singlet scalar \check{x}_8 is dualized to a U(1) gauge field by replacing

$$\partial_\mu\check{x}_8 = \frac{1}{2g_{\text{YM}}}\epsilon_{\mu\nu\rho}\check{F}^{\nu\rho}, \quad (3.18)$$

where $\check{F}_{\nu\lambda}$ is a U(1) gauge field strength. The action for the scalar fields S_X (2.5) is now given by

$$S_X = \int d^3x \text{Tr} \left[-\tilde{D}_\mu \tilde{X}_i \tilde{D}^\mu \tilde{X}_i - \frac{1}{2g_{\text{YM}}^2} \check{F}_{\mu\nu} \check{F}^{\mu\nu} + \frac{1}{2g_{\text{YM}}^2} \left(F_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[B_{\mu\nu}] \right)^2 - V_{\text{bos}} \right] + \mathcal{O}\left(\frac{1}{v}\right), \quad (3.19)$$

while the Chern-Simons action (2.6) is given by

$$S_{\text{CS}} = \int d^3x \text{Tr} \left[-\frac{1}{g_{\text{YM}}^2} \left(F_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[B_{\mu\nu}] \right) F^{\mu\nu} \right] + \mathcal{O}\left(\frac{1}{v}\right). \quad (3.20)$$

For the R-R three-form fields in ten dimensions, we define the U(2) adjoint antisymmetric three-form fields by using (3.12),

$$\tilde{C}_{PRS} = \frac{1}{2} [C_{PRS} + C_{PRS}^\dagger - i(C_{PRS} - C_{PRS}^\dagger)], \quad (3.21)$$

where we choose 1/2 as a normalization different from the case of NS-NS two-form field. Then we have the $C_{(3)}$ part of the WZ-type action S_C ,

$$S_C^{(3)} = \int d^3x \epsilon^{\mu\nu\rho} \times \text{Tr} \left\{ \frac{\mu_2}{3!} \left[\tilde{C}_{\mu\nu\rho} + 3\lambda \tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\lambda^2 \tilde{C}_{\mu ij} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j + \lambda^3 \tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k \right] + \mu_2 \lambda \left(v P[B_{\mu\nu}] A_\rho^- + P[\check{B}_{\mu\nu}] \partial_\rho \check{x}_8 \right) \right\} + \mathcal{O}\left(\frac{1}{v}\right), \quad (3.22)$$

where $P[\check{B}_{\mu\nu}]$ is the trace part $P[\tilde{B}_{\mu\nu}]$. Substituting (3.17)–(3.18) into (3.22) for A_μ^- and $\partial_\rho \check{x}_8$ and taking into account the constants given in (3.16), we obtain

$$S_C^{(3)} = \int d^3x \text{Tr} \times \left\{ \frac{\mu_2}{3!} \epsilon^{\mu\nu\rho} \left[\tilde{C}_{\mu\nu\rho} + 3\lambda \tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\lambda^2 \tilde{C}_{\mu ij} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j + \lambda^3 \tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k \right] - \frac{1}{g_{\text{YM}}^2 \tilde{\lambda}} \left(\lambda P[B_{\mu\nu}] (F^{\mu\nu} + \frac{1}{\tilde{\lambda}} P[B^{\mu\nu}]) + \lambda P[\check{B}_{\mu\nu}] \check{F}^{\mu\nu} \right) \right\} + \mathcal{O}\left(\frac{1}{v}\right). \quad (3.23)$$

In order to match the mass dimension of the ten-dimensional transverse scalar fields, we rescale the scalar fields as $\tilde{X}_i \rightarrow \frac{\tilde{X}_i}{g_{\text{YM}}}$. Applying this rescaling to (3.19), (3.20), (3.23) and summing them, we get

$$S_X + S_{\text{CS}} + S_C^{(3)} = \int d^3x \text{Tr} \left\{ \frac{1}{g_{\text{YM}}^2} \left[-\tilde{D}_\mu \tilde{X}_i \tilde{D}^\mu \tilde{X}_i - \frac{1}{2} \left(\check{F}_{\mu\nu} \check{F}^{\mu\nu} + \frac{2}{\tilde{\lambda}} \check{F}_{\mu\nu} P[\check{B}^{\mu\nu}] \right) - \frac{1}{2} \left(F_{\mu\nu} + \frac{1}{\tilde{\lambda}} P[B_{\mu\nu}] \right) \left(F^{\mu\nu} + \frac{1}{\tilde{\lambda}} P[B^{\mu\nu}] \right) - \frac{1}{2} [\tilde{X}_i, \tilde{X}_j] [\tilde{X}_i, \tilde{X}_j] \right] + \frac{\mu_2}{3!} \epsilon^{\mu\nu\rho} \left[\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda} \tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\tilde{\lambda}^2 \tilde{C}_{\mu ij} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j + \tilde{\lambda}^3 \tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k \right] \right\}, \quad (3.24)$$

where we used the explicit form of the scalar potential V_{bos} in ref. [7].

Next we turn to the $C_{(6)}$ part of S_C . After the aforementioned rescaling $\tilde{X}_i \rightarrow \frac{\tilde{X}_i}{g_{\text{YM}}}$, the Higgs rule for X_{ij8} [8] becomes

$$X_{ij8} \rightarrow \frac{v}{4g_{\text{YM}}^2} [\tilde{X}_i, \tilde{X}_j], \quad X_{ij8}^\dagger \rightarrow -\frac{v}{4g_{\text{YM}}^2} [\tilde{X}_i, \tilde{X}_j]. \quad (3.25)$$

According to (3.21), we introduce the U(2) adjoint antisymmetric six-form fields,

$$\tilde{C}_{PQRSTV} = \frac{1}{2} \left[C_{PQRSTV} + C_{PQRSTV}^\dagger - i(C_{PQRSTV} - C_{PQRSTV}^\dagger) \right], \quad (3.26)$$

and identify the R-R five-form fields in ten-dimensional type II string theory as

$$\tilde{C}_{PQRST} = \tilde{C}_{PQRST8}. \quad (3.27)$$

The first term in the $C_{(6)}$ Lagrangian in (3.10) is given by

$$\begin{aligned} & \frac{\mu'_2}{2} \text{Tr} \left(C_{\mu\nu\rho IJK} X_{IJK}^\dagger + C_{\mu\nu\rho IJK}^\dagger X_{IJK} \right) \\ &= -i \frac{3v\mu'_2}{8g_{\text{YM}}^2} \text{Tr} \left\{ -i(C_{\mu\nu\rho ij8} - C_{\mu\nu\rho ij8}^\dagger) [\tilde{X}_i, \tilde{X}_j] \right\} + \mathcal{O}\left(\frac{1}{v}\right). \end{aligned} \quad (3.28)$$

Since $[\tilde{X}_i, \tilde{X}_j]$ is traceless, the product $(C_{\mu\nu\rho ij8} + C_{\mu\nu\rho ij8}^\dagger) [\tilde{X}_i, \tilde{X}_j]$ is traceless. Therefore, we can freely include this term in the last equation (3.28) to get

$$\begin{aligned} & \frac{\mu'_2}{2} \text{Tr} \left(C_{\mu\nu\rho IJK} X_{IJK}^\dagger + C_{\mu\nu\rho IJK}^\dagger X_{IJK} \right) \\ &= -i \frac{3v\mu'_2}{8g_{\text{YM}}^2} \text{Tr} \left(\tilde{C}_{\mu\nu\rho ij8} [\tilde{X}_i, \tilde{X}_j] \right) = -i \frac{\mu_2 \tilde{\lambda}}{2} \text{Tr} \left(\tilde{C}_{\mu\nu\rho ij} [\tilde{X}_i, \tilde{X}_j] \right). \end{aligned} \quad (3.29)$$

In the second equality we have used $\mu'_2 = \beta\lambda\mu_2$ and have chosen $\beta = \frac{4\pi}{3k}$ in order to match the coefficient with the coefficient of the corresponding term in type IIA string theory in (2.3). Following the same procedure, we can calculate the remaining terms in the $C_{(6)}$ Lagrangian in (3.10),

$$\begin{aligned} & \frac{3}{2} \mu'_2 \lambda \text{Tr} \left(C_{\mu\nu LIJK} \langle\langle X_{IJK}^\dagger D_\rho X_L \rangle\rangle \right) + \text{c.c.} \\ &= -\frac{3i}{2} \mu_2 \tilde{\lambda}^2 \text{Tr} \left(\tilde{C}_{\mu\nu jk} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\rho \tilde{X}_k \rangle\rangle \right), \end{aligned} \quad (3.30)$$

$$\begin{aligned} & \frac{3}{2} \mu'_2 \lambda^2 \text{Tr} \left(C_{\mu LMIJK} \langle\langle X_{IJK}^\dagger D_\nu X_L (D_\rho X_M)^\dagger \rangle\rangle \right) + \text{c.c.} \\ &= -\frac{3i}{2} \mu_2 \tilde{\lambda}^3 \text{Tr} \left(\tilde{C}_{\mu ijkl} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\nu \tilde{X}_k \tilde{D}_\rho \tilde{X}_l \rangle\rangle \right), \end{aligned} \quad (3.31)$$

$$\begin{aligned} & \frac{1}{2} \mu'_2 \lambda^3 \text{Tr} \left(C_{LMNIJK} \langle\langle X_{IJK}^\dagger D_\mu X_L (D_\nu X_M)^\dagger D_\rho X_N \rangle\rangle \right) + \text{c.c.} \\ &= -\frac{i}{2} \mu_2 \tilde{\lambda}^4 \text{Tr} \left(\tilde{C}_{ijklm} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\mu \tilde{X}_k \tilde{D}_\nu \tilde{X}_l \tilde{D}_\rho \tilde{X}_m \rangle\rangle \right). \end{aligned} \quad (3.32)$$

Summing (3.24) and (3.29)–(3.32), we finally reach

$$\begin{aligned}
S_{\text{tot}} = \int d^3x \text{Tr} \left\{ \frac{1}{g_{\text{YM}}^2} \left[-\tilde{D}_\mu \tilde{X}_i \tilde{D}^\mu \tilde{X}_i - \frac{1}{2} \left(\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2}{\lambda} \tilde{F}_{\mu\nu} P[\tilde{B}^{\mu\nu}] \right) \right. \right. \\
- \frac{1}{2} \left(F_{\mu\nu} + \frac{1}{\lambda} P[B_{\mu\nu}] \right) \left(F^{\mu\nu} + \frac{1}{\lambda} P[B^{\mu\nu}] \right) - \frac{1}{2} [\tilde{X}_i, \tilde{X}_j] [\tilde{X}_i, \tilde{X}_j] \left. \right] \\
+ \frac{\mu_2}{3!} \epsilon^{\mu\nu\rho} \left[\tilde{C}_{\mu\nu\rho} + 3\tilde{\lambda} \tilde{C}_{\mu\nu i} \tilde{D}_\rho \tilde{X}_i + 3\tilde{\lambda}^2 \tilde{C}_{\mu i j} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j + \tilde{\lambda}^3 \tilde{C}_{i j k} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k \right] \\
+ \frac{\mu_2}{3!} \epsilon^{\mu\nu\rho} \left[-\frac{i}{2} \tilde{\lambda} \tilde{C}_{\mu\nu\rho i j} 8 [\tilde{X}_i, \tilde{X}_j] - \frac{3i}{2} \tilde{\lambda}^2 \tilde{C}_{\mu\nu i j k} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\rho \tilde{X}_k \rangle\rangle \right. \\
\left. - \frac{3i}{2} \tilde{\lambda}^3 \tilde{C}_{\mu i j k l} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\nu \tilde{X}_k \tilde{D}_\rho \tilde{X}_l \rangle\rangle - \frac{i}{2} \tilde{\lambda}^4 \tilde{C}_{i j k l m} \langle\langle [\tilde{X}_i, \tilde{X}_j] \tilde{D}_\mu \tilde{X}_k \tilde{D}_\nu \tilde{X}_l \tilde{D}_\rho \tilde{X}_m \rangle\rangle \right] \left. \right\}. \tag{3.33}
\end{aligned}$$

The second and third terms in (3.33) are unified to form the kinetic term of U(2) gauge field, as the gauge invariant combination on the world-volume of D-brane,

$$-\frac{1}{2g_{\text{YM}}^2} \text{Tr} \left[\left(\tilde{F}_{\mu\nu} + \frac{1}{\lambda} P[\tilde{B}_{\mu\nu}] \right) \left(\tilde{F}^{\mu\nu} + \frac{1}{\lambda} P[\tilde{B}^{\mu\nu}] \right) \right] \tag{3.34}$$

up to the quadratic term in $P[\tilde{B}_{\mu\nu}]$ which belongs to nonlinear terms in $C^{(3)}$. Here we notice that in addition to the natural couplings of the D2-brane to the three-form field and the dual five-form field in type IIA superstring theory, the WZ-type action also produces the coupling between $\tilde{F}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ in the linearized nonabelian DBI action for D2-brane. Unfortunately, however, the dimensional reduction of the WZ-type action (2.8) does not produce $\tilde{C}^{(3)} \wedge \tilde{F}$ and $\tilde{C}^{(5)} \wedge \tilde{F}$ -terms which appear in ten dimensional WZ-type action (2.3). We need more investigations in this direction.

Our derivation of the result in (3.33) is based on the elegant Higgs rule of [8]. Here we would like to comment on a mild problem in applying these rules. We know that the transverse scalars X_i are bi-fundamentals of $SU(2) \times SU(2)$. Therefore, to obtain the U(2) adjoint scalar, the trace and the traceless part of these scalars should be combined as $\tilde{X}_i = \check{x}_i + \mathbf{x}_i$. A similar rewriting should also be made for the form fields. When we are dealing with the BLG theory without WZ-type coupling, the Higgs rule of [8] are exactly the net effect of this splitting and recombination of the trace and traceless part of the fields. However, in the presence of the WZ-type coupling containing more than two covariant derivatives, the splitting and recombination of the trace and traceless parts reexpress most of the terms of S_C in terms of the U(2) adjoint fields except a few terms which lead to some mismatch. To demonstrate this observation we consider the C_{ijk} -term,

$$\begin{aligned}
\frac{1}{2} \epsilon^{\mu\nu\rho} \text{Tr} \left[C_{ijk} (\tilde{D}_\mu X_i)^\dagger \tilde{D}_\nu X_j (\tilde{D}_\rho X_k)^\dagger + C_{ijk}^\dagger \tilde{D}_\mu X_i (\tilde{D}_\nu X_j)^\dagger \tilde{D}_\rho X_k \right] \\
= \epsilon^{\mu\nu\rho} \text{Tr} \left(\tilde{C}_{ijk} \tilde{D}_\mu \tilde{X}_i \tilde{D}_\nu \tilde{X}_j \tilde{D}_\rho \tilde{X}_k - 2\tilde{C}_{ijk} \tilde{D}_\mu \check{x}_i \tilde{D}_\nu \check{x}_j \tilde{D}_\rho x_k - 2\tilde{C}_{ijk} \tilde{D}_\mu \check{x}_i \tilde{D}_\nu x_j \tilde{D}_\rho x_k \right), \tag{3.35}
\end{aligned}$$

where the covariant derivative is given in (3.3). We have also made the following splitting and recombination of the trace and traceless part of the three-form field

$$C_{ijk} = \frac{1}{2} \tilde{c}_{ijk} \mathbf{1} + i c_{ijk}^\alpha \frac{\sigma^\alpha}{2}, \quad C_{ijk}^\dagger = \frac{1}{2} \tilde{c}_{ijk} \mathbf{1} - i c_{ijk}^\alpha \frac{\sigma^\alpha}{2}, \quad \tilde{C}_{ijk} = \frac{1}{2} \tilde{c}_{ijk} \mathbf{1} + c_{ijk}^\alpha \frac{\sigma^\alpha}{2}. \tag{3.36}$$

Note that the the last two terms in (3.35) cannot entirely be expressed in terms of the U(2) adjoint fields. This mismatch is generated from the cross terms between the trace and traceless sectors. It is quite straightforward to show that, in the U(1)×U(1) ABJM theory, there is no such mismatch. We will leave verification of the absence of such mismatch in ABJM theory with arbitrary gauge group for the future work [19].

4 Quartic mass-deformation term from a C₍₆₎ term

Let us recall the bosonic part of supersymmetry-preserving mass-deformation terms in the BLG theory [13, 14] in order to compare these with the WZ-type action in (2.9),

$$\begin{aligned}
 S_m = & \int d^3x m^2 \text{Tr} \left(X_I X_I^\dagger + X_I^\dagger X_I \right) \\
 & + \frac{4\pi m}{k} \int d^3x \text{Tr} \left[X_3(X_4)^\dagger X_5(X_6)^\dagger - X_5(X_4)^\dagger X_3(X_6)^\dagger \right. \\
 & \quad \left. + X_7(X_8)^\dagger X_9(X_{10})^\dagger - X_9(X_8)^\dagger X_7(X_{10})^\dagger \right], \tag{4.1}
 \end{aligned}$$

where m is the mass parameter. According to [12], this mass term comes from the background four-form flux which is (anti-)self-dual in eight-dimensional transverse space. We examine specifically how (4.1) can appear from WZ-type coupling (2.8). Due to the (anti-)self-dual property of the flux, we should consider the contribution from both four-form $F_{(4)}$ and dual seven-form $F_{(7)}$. Let us first take into account the contribution from $F_{(7)}$ by turning on only specific components of $F_{(7)}$ as

$$F_{\mu\nu\rho IJKL} = \frac{\beta m}{\lambda \mu_2'} \epsilon_{\mu\nu\rho} T_{IJKL}, \tag{4.2}$$

where T_{IJKL} is (anti-)self-dual in eight-dimensional transverse space

$$T^{IJKL} = \pm \frac{1}{4!} \epsilon^{IJKL I' J' K' L'} T_{I' J' K' L'}. \tag{4.3}$$

The corresponding WZ-type action of our consideration (2.9) is

$$S_C^{(6)} = \int dx^\mu \wedge dx^\nu \wedge dx^\rho \frac{\mu_2'}{2} \text{Tr} \left[C_{\mu\nu\rho IJK} (X_I)^\dagger X_J (X_K)^\dagger + (\text{c.c.}) \right] + \dots \tag{4.4}$$

When the six-form $C_{\mu\nu\rho IJK}$ is the potential of constant seven-form field strength $F_{\mu\nu\rho IJKL}$, we obtain it explicitly,

$$C_{\mu\nu\rho IJK} = \lambda F_{\mu\nu\rho IJKL} X_L = \frac{\beta m}{\mu_2'} \epsilon_{\mu\nu\rho} T_{IJKL} X_L. \tag{4.5}$$

Substituting the six-form field configuration (4.5) with (4.3) into the WZ-type action (4.4), we have

$$\begin{aligned}
 S_C^{(6)} &= \frac{4\pi m}{3k} \int dx^\mu \wedge dx^\nu \wedge dx^\rho \epsilon_{\mu\nu\rho} \frac{1}{2} \text{Tr} \left[T_{IJKL} X_L (X_I)^\dagger X_J (X_K)^\dagger + (\text{c.c.}) \right] + \dots \\
 &= -\frac{4\pi m}{k} \int d^3x \text{Tr} \left[T_{IJKL} X_L (X_I)^\dagger X_J (X_K)^\dagger + (\text{c.c.}) \right] + \dots, \tag{4.6}
 \end{aligned}$$

which exactly coincides with the quartic mass-deformation term in (4.1) as far as the four-form tensor T_{IJKL} satisfies

$$T_{1234} = T_{5678} = 1, \quad \text{other independent components} = 0. \quad (4.7)$$

This configuration keeps the maximal supersymmetry $\mathcal{N} = 8$, and turning on other components of T_{IJKL} in addition to (4.7) leads to less supersymmetry. The relation between nonzero components of T_{IJKL} and the number of supersymmetry has been studied in the context of field theory [14, 20, 21] and in the dual AdS side [12].

The contribution from the four-form tensor is also calculated in a similar way. From (4.2) the configuration of $F_{(4)}$ is

$$F_{IJKL} \sim \frac{\beta m}{\lambda \mu_2} T_{IJKL} = \frac{m}{\lambda^2 \mu_2} T_{IJKL}. \quad (4.8)$$

Then the corresponding WZ-type action is

$$\begin{aligned} S_C^{(3)} &\sim \mu_2 \int dx^\mu \wedge dx^\nu \wedge dx^\rho \text{Tr} \left[\frac{m}{\lambda^2 \mu_2} T_{IJKL} \lambda^4 X_L (D_\mu X_I)^\dagger D_\nu X_J (D_\rho X_K)^\dagger + (\text{c.c.}) \right] + \dots \\ &= \lambda^2 m \int dx^\mu \wedge dx^\nu \wedge dx^\rho \text{Tr} \left[T_{IJKL} X_L (D_\mu X_I)^\dagger D_\nu X_J (D_\rho X_K)^\dagger + (\text{c.c.}) \right] + \dots \end{aligned} \quad (4.9)$$

Since $\lambda^2 \sim l_p^3$, this term does not contribute under the limit $l_p \rightarrow 0$.

We confirm the identification of a cubic WZ-type term (4.4) with a specific form of constant (anti-)self-dual four-form flux (4.2) in Minkowski spacetime signature. If we take a Euclideanization to the flux (4.2), then an overall imaginary number i appears in the left-hand side. It implies that μ in (4.2) may not be a mass parameter but a chemical potential.

For the quadratic mass term which can be interpreted as the quadratic coupling of form fields between M2's, we do not have natural argument to fix it. In the case of string theory, this coupling is obtained from the world-sheet disk amplitude with insertion of the two R-R vertex operators. Though we basically have ambiguity for the position of two insertions, we can avoid this ambiguity by introducing appropriate auxiliary fields [22, 23] and compute the coupling at least for some particular cases [24, 25].

5 Conclusion and discussion

Once the world-volume action of N stacked M2-branes is determined, it is interesting to understand how the M2-branes couple to the bulk fields. In this paper, we constructed the WZ-type action which describes the coupling of the M2-branes to antisymmetric three- and six-form fields in M-theory. We consider the BLG theory for two M2-branes and write down a WZ-type action linear to antisymmetric three- and six-form fields in analogy with the corresponding action in type IIA string theory. When it reduces to ten dimensions through a circle compactification, our action reproduces the expected ten-dimensional coupling of R-R and NS-NS form fields to D2-branes in type IIA string theory.

In addition to our main goal of obtaining the WZ-type coupling of M2-branes, we show that a particular cubic WZ-type term can be identified with the quartic scalar interaction in the supersymmetry-preserving mass deformation of the BLG theory. We made this identification in a flat world-volume and transverse space by making an assumption that the seven-form field strength is constant and is proportional to the mass parameter of mass deformation term.

A few discussions are in order. Though the ten-dimensional WZ-type action (2.1) is restricted to the terms linear in $\tilde{C}_{(n)}$, it contains $\tilde{C}_{(n)} \wedge e^{\tilde{B}}$ where \tilde{B} is NS-NS two-form field. Since the NS-NS two-form field in string theory comes from a part of $C_{(3)}$ in M-theory, inclusion of quadratic or higher order terms in $C_{(3)}$ in addition to the WZ-type action (2.9) seems natural. For instance the quadratic term in $C_{(3)}$ is

$$\begin{aligned}
 S_{C^2} &= \frac{\mu_2''}{2} \int_{2+1} \text{Tr} \left(P[\langle i_X i_X i_X \rangle C_{(3)} \wedge C_{(3)}] \right) \\
 &= 5\mu_2'' \int \frac{1}{3!} d^3 x \epsilon^{\mu\nu\rho} \text{Tr} \left[\frac{1}{2} \hat{C}_{[\mu\nu\rho]} \langle\langle C_{IJK}^\dagger X_{IJK} \rangle\rangle + \frac{1}{2} C_{[\mu\nu\rho]} \langle\langle C_{IJK} X_{IJK}^\dagger \rangle\rangle \right. \\
 &\quad + 3\lambda C_{[\mu\nu L]} \langle\langle C_{IJK}^\dagger X_{IJK} (D_\rho X_L)^\dagger \rangle\rangle \\
 &\quad + \frac{3}{2} \lambda^2 \hat{C}_{[\mu LM]} \langle\langle C_{IJK}^\dagger X_{IJK} (D_\nu X_L)^\dagger D_\rho X_M \rangle\rangle \\
 &\quad + \frac{3}{2} \lambda^2 C_{[\mu LM]} \langle\langle C_{IJK} X_{IJK}^\dagger D_\nu X_L (D_\rho X_M)^\dagger \rangle\rangle \\
 &\quad \left. + \lambda^3 C_{[LMN]} \langle\langle C_{IJK}^\dagger X_{IJK} (D_\mu X_L)^\dagger D_\nu X_M (D_\rho X_N)^\dagger \rangle\rangle + (\text{c.c.}) \right], \tag{5.1}
 \end{aligned}$$

where $\mu_2'' = \beta' \lambda \mu_2$. Like β , the value of β' is also determined by comparing this term with an appropriate term in the corresponding action of type IIA superstring theory, after a circle compactification. Specifically, when $\beta' = \beta = 4\pi/3k$, this term exactly coincides with the $C_{(3)} \wedge B$ term in (2.1).

In this paper we constructed the bosonic sector of WZ-type action coupled to the world-volume fields of M2-branes. If we supersymmetrize what we obtained, then we may reach the supersymmetric WZ-type action in M-theory. Though we fixed the coefficient of the WZ-type action (2.9) by comparing with the terms of the ten-dimensional R-R coupling action (2.1) through the compactification of the eighth transverse direction, this indirect fixation procedure can be reconfirmed by constructing the supersymmetric WZ-type action coupled to M2-branes. Then, this understanding will also help the extension to general case of arbitrary number of stacked multiple M2-branes of which the world-volume theory is described by the $\mathcal{N} = 6$ superconformal Chern-Simons gauge theory with $U(N) \times U(N)$ gauge symmetry. We will report the construction of WZ-type coupling in the context of ABJM theory for arbitrary number of M2-branes in the subsequent work [19].

In the BLG and ABJM theories, M2-branes and $\bar{M}2$ -branes are not distinguished as the case of DBI type world-volume action of D-branes. The D- and \bar{D} -branes carry opposite sign R-R charges and are distinguished by the R-R coupling (2.1) in type II string theories [3]. Similarly the M2- and $\bar{M}2$ -branes are also distinguishable by the analogue of WZ-type in

the M-theory (2.9). This will also let the construction of world-volume action of $M2\bar{M}2$ pair without supersymmetry tractable.

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